Online supplementary material to:

Intra- and inter-industry misallocation and comparative advantage

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In this supplementary material I explore the relative importance of inter-industry factor misallocation in a closed economy with a fixed number of operating firms, following the standard specification of Hsieh and Klenow (2009). Section 1 presents the formulas for the TFP gains from removing each type of misallocation and computes the corresponding magnitudes for two countries, Colombia and China, using different industry clasifications. Section 2 shows robustness exercises. Section 3 evaluates whether inter-industry misallocation can also explain TFP gaps across countries.

Through this material, I follow the same notation as in Section 2 of the paper.

1 Intra- and inter-industry misallocation

Denote the TFPQ and TFPR of firm producing variety m as a_m and ψ_m, respectively, and ξ_{lm} the MRP of the input l. Let $\bar{\xi}_{ls}$ denote the HWA of ξ_{lm}, with weights given by the participations of firm's revenues in total industry revenue. Note that $\bar{\xi}_{ls} = (1+\bar{\theta}_{ls})\frac{w_l}{\rho}$. Using the cost minimization condition of the CD aggregator across sectors, total demand of factor-l in industry s can be expressed as:

$$Z_{ls} = \frac{\alpha_{ls}\beta_s/\bar{\xi}_{ls}}{\sum\limits_{s}^{S}\alpha_{ls}\beta_s/\bar{\xi}_{ls}}\bar{Z}_l \tag{1}$$

where $\bar{Z}_l \equiv \sum\limits_{s}^{S} Z_{ls}$ correspond to the fixed endowment of factor-l in the economy. Standard aggregation under monopolistic competition leads to an industry production of the form $Q_s = A_s M_s^{\frac{1}{\sigma-1}} \prod\limits_{l}^{L} Z_{ls}{}^{\alpha_{ls}}$, where sectoral TFP A_s can be derived from firm-level data from:

$$A_s^{\sigma-1} = \frac{1}{M_s}\sum_{m}^{M_s}\left(\frac{a_m\bar{\psi}_s}{\psi_m}\right)^{\sigma-1} \tag{2}$$

where $\bar{\psi}_s$ is the sectoral revenue productivity. If a reform equalizes TFPR across firms, the sectoral (efficient) TFP is simply the power mean of physical productivities: $\widetilde{A}_s^{\sigma-1} = \tilde{M}_s^{-1}\sum\limits_{m}^{\tilde{M}_s} a_m^{\sigma-1}$. With the assumption of no self-selection of firms, $\tilde{M}_s = M_s$ and the percentage gains on sectoral TFP due to TFPR equalization are:

$$Gains_s^{intra} = 100(\frac{\widetilde{A}_s}{A_s} - 1) = 100((\sum_{m}^{M_s}(\frac{a_m\bar{\psi}_s}{\widetilde{A}_s\psi_m})^{\sigma-1})^{\frac{1}{1-\sigma}} - 1) \tag{3}$$

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Equation (3) is the cornerstone of HK's counterfactual exercise, and the description until here provided summarizes the main features of HK's model. The gains from removing intra-industry misallocation in (3) are the same if the reform equalizes firms' TFPR to $\bar{\psi}_s$, so the factors' MRP are equal to their HWA in the industry, or to the inter-industry efficient allocation, in which case the factors' MRP are equated to $\frac{w_l}{\rho}$. However, only in the first case it is ensured there are no factor reallocations across sectors (which is evident from equation 1), so the sectoral TFP gains in equation (3) are identical to the gains in industry output, $100(\frac{\widetilde{Q}_s}{Q_s} - 1)$. In this specific case, total output gains in the economy can be computed simply by aggregating sectoral productivities up using the CD aggregator across industries:

$$Gains^{intra} = 100(\prod_s^S (\frac{\widetilde{A}_s}{A_s})^{\beta_s} - 1) \tag{4}$$

Clearly, total gains in (4) are only due to resource reallocation within industries: by assumption, there are not factor reallocations across sectors. In this case, there is MRP equalization within industries, but not necessarily across them. In the more general case in which I impose MRP equalization not only within but across industries (i.e. removing all wedges), sectoral TFP gains are the same as in (3), but output gains in each industry are no longer equal to the corresponding TFP gains, due to factor reallocation across sectors. From (1), the allocative efficient demand of factors at the industry level is given by $\widetilde{Z}_{ls} = \alpha_{ls}\beta_s\bar{Z}_l/\sum_s^S \alpha_{ls}\beta_s$.[1] Industry's output in frictionless factor markets is given by $\tilde{Q}_s = \tilde{A}_s \tilde{M}_s^{\frac{1}{\sigma-1}} \prod_l^L \tilde{Z}_{ls}{}^{\alpha_{ls}}$. Thus, the variation in sectoral output due to a reform that removes all wedges is a consequence of both a rise in the TFP and a variation in the use of factors in the whole sector, which depends exclusively on the sign of $\bar{\theta}_{ls}$ (the extent of inter-industry misallocation). At the aggregate level, factor endowments between the distorted economy and the allocative efficient counterfactual are kept constant. So any change in aggregate output Q is attributable to variations in the aggregate TFP, and it is due to resource reallocation, both within and between industries. Gains in aggregate TFP can be caused by increases in sectoral TFP, term denoted $Gains^{intra}$ above, or by reallocation of factors between industries, given by:

$$Gains^{inter} = 100(\prod_s^S \prod_l^L \frac{\widetilde{Z}_{ls}{}^{\alpha_{ls}\beta_s}}{Z_{ls}{}^{\alpha_{ls}\beta_s}} - 1) = 100(\prod_s^S \prod_l^l [\frac{\sum_s^S (\alpha_{ls}\beta_s/\bar{\xi}_{ls})}{(\sum_s^S \alpha_{ls}\beta_s)/\bar{\xi}_{ls}}]^{\alpha_{ls}\beta_s} - 1) \tag{5}$$

Where I use equation (1) and the expression for $\widetilde{Z}_{ls}$ to obtain the explicit closed-form solution. Thus, inter-industry gains only depend on the industry average MRP interacted with technological parameters, a plain consequence of the sectoral demand of factors in equation (1). These gains can be computed only with industry-level data, a fact that allows me to make cross-country comparisons to evaluate whether this component also explains the TFP gaps observed across countries, an exercise that is performed below. Finally, total gains in the economy, given by the variation on total output (or aggregate TFP), are a combination of both sources of gains:

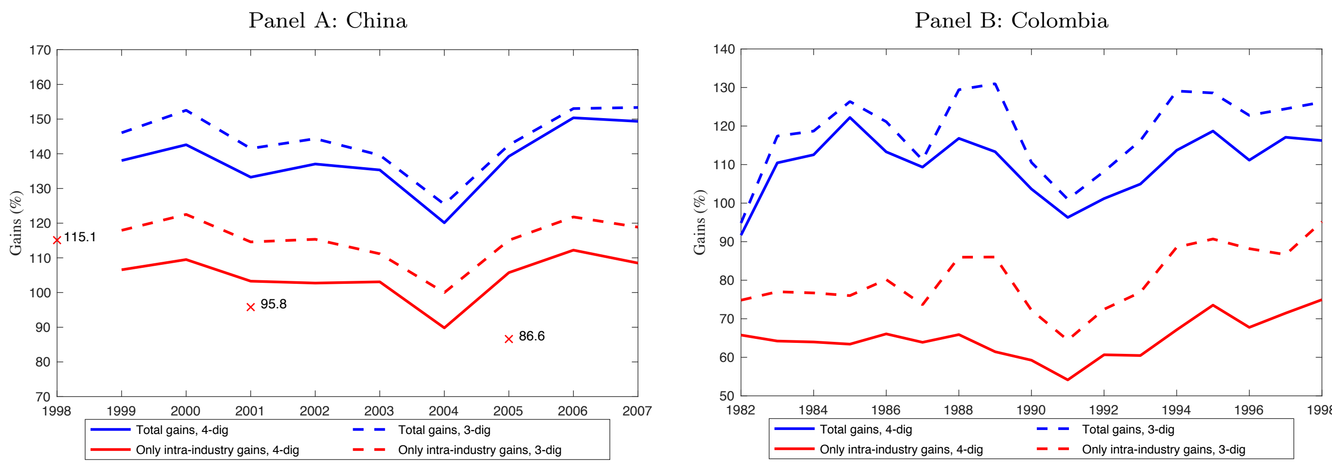
$$Gains = 100(\frac{\widetilde{Y}}{Y} - 1) = 100[(\frac{Gains^{inter}}{100} + 1)(\frac{Gains^{intra}}{100} + 1) - 1] \tag{6}$$

The importance of each type of misallocation depends, of course, on the considered industry aggregation. For example, in the extreme case in which the whole manufacturing sector is represented as a single industry, the entire TFP loss due to allocative inefficiency proceeds from the intra-industry

[1] This is, in the case that all sectors have the same revenue shares, the efficient allocation of factors across sectors implies that more intensive industries should have a larger proportion of the corresponding factor. Similarly, in the case that all sectors have the same factor intensities, the factors should be allocated in proportion only on sectoral revenue shares. The efficient factor allocation across industries is the combination of these two forces.

type, whereas in the opposite extreme, the whole loss proceeds from the inter-sectoral type. Using a 4-digit ISIC industry classification,[2] a value added specification for the production function, and average US cost shares at the corresponding aggregation level from the NBER-CES Manufacturing Industry Database during the same period, the same set of specifications than the used in HK's baseline, I find that the inter-sectoral component contributes on average up to 35% of the total reallocation gains of a comprehensive reform that removes all factor misallocation in Colombia, for the period 1982-1998. As a robustness check, I replicate the exercise with firm-level data from China, a country that offers external validation using the calculations provided by HK.[3] In Figure 1 I report using continue lines the total gains (blue) and the intra-sectoral gains (red) from removing distortions for both countries, when the 4-digit ISIC industry aggregation is used. The difference between both lines is due to the gains from inter-sectoral reallocation. For China I find similar TFP gains as in HK in the case of removing only intra-industry misallocation, and an average contribution of 30% of the inter-sectoral component for the complete reform.

Figure 1 – TFP gains from factor reallocation in a closed economy



Note: In Panel A, × correspond to the values found by HK.

In general, gains from removing distortions are larger for China, although the time periods are not comparable. The graph shows that over time in both countries there are not significant improvements in allocative efficiency in the considered periods; indeed, there is a slight worsening at the end of each one. When I move to the 3-digit ISIC classification, the predictions from the decomposition seem to hold. The dashed lines in Figure 1 report once again the total gains (blue) and the intra-sectoral gains (red) from removing distortions, but now at the 3-digit ISIC classification. Both total gains fluctuate around a similar range. However, the intra-industry gains rise in a larger proportion than the total gains, so their average contribution is now 68% and 73% for Colombia and China, respectively. This confirms that as the level of disaggregation increases, the intra-industry gains are lower.

2 Robustness checks

The source of inter-industry gains is neither related to the use of US cost shares instead of domestic factor intensities in the sectoral production function nor to the use of a value-added specification. For

[2]For the 4-digit classification in the Colombian case, due to small number of observations, 14 industries were reclassified to its closest 4-digit industry or to the 4-digit sector within the same 3-digit industry that merges the products not elsewhere classified.

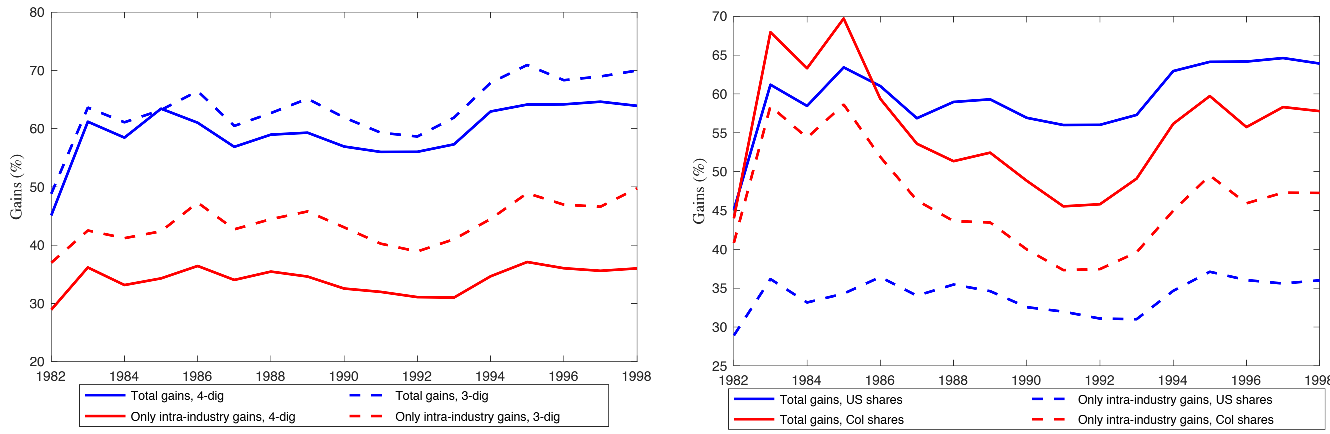
[3]For China, I use the panel from the Annual Survey of Industrial Production collected by the Chinese government's National Bureau of Statistics, for the period 1999-2007

example, Figure 2 displays for the Colombian case that using a gross-output specification (Panel A) or changing the production function coefficients for Colombian cost shares (Panel B) does not alter importantly the key insights. In the latter case, factor intensities are now equal to the observed share costs, but they are still different to the optimal share cost in monopolistic competition (where the total cost is ρ times the revenue), which is what matters in the efficient allocation. However, the use of Colombian cost shares reduces the relative importance of inter-sectoral reallocation: its average contribution shrinks to 23%.

Figure 2 – Sensitivity to production function specification and factor intensities

Panel A : TFP gains using gross output specification (Colombia, US cost shares)

Panel B : TFP gains by set of cost shares (Colombia, 4-dig, gross output specification)



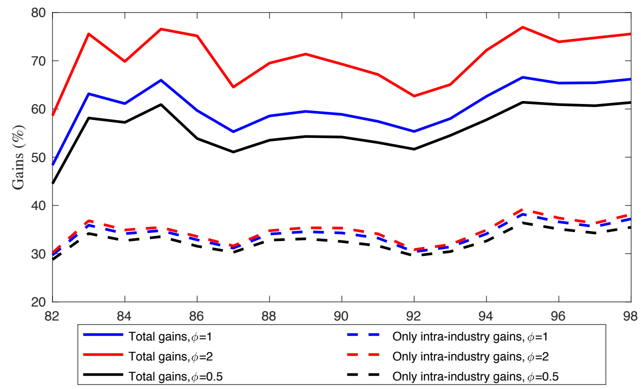
Further, the total gains and the contribution of the inter-sectoral component increase using a higher elasticity of substitution across sectors. This is completely in line with the HK prediction that when sectors outputs are better substitutes, inputs are reallocated toward sectors with bigger productivity gains, so there are larger TFP gains. We can show this with a CES demand across sectors. In this case, there is not a closed-form solution for each component, but it is possible to implement a numerical procedure to obtain both gains[4]. Figure 3 shows that for different values of the elasticity of substitution across sectors (ϕ), the components of the gains behave as predicted. The numerical procedure replicates the results of the close-form solutions for the CD aggregator for both components in the case $\phi = 1$, whereas total gains and the contribution of the inter-sectoral component increases when $\phi = 2$ (up to 50% from 43% in the latter case) and decreases when $\phi = 0.5$ (to 36% in the latter case). In those exercises the change in the intra-sectoral gains is negligible.

[4]With a CES aggregator of the form $Y^{\varphi} = \sum_{s}^{S} \beta_s {Y_s}^{\varphi}$, where $\varphi = \frac{\phi - 1}{\phi}$ and ϕ is the elasticity of substitution across sectors, the sectoral factor demand is now:

$$Z_{ls} = \frac{\alpha_{ls} \beta_s^{\phi} P_s^{1-\phi} / \bar{\xi}_{ls}}{\sum_{s}^{S} \alpha_{ls} \beta_s^{\phi} P_s^{1-\phi} / \bar{\xi}_{ls}} \bar{Z}_l$$

Thus, in the allocative efficient inter-industry allocation, not only factor intensities and revenue shares play a role, but also the efficient sectoral price indexes as indicators of productivity. The direction and strength of their influence depends on the magnitude of ϕ. For $\phi > 1$ ($\phi < 1$), if factor intensities and shares of sectoral revenue are constant across sectors, factors should be allocated to more (less) productive sectors. The interaction of these three sectoral forces (factor intensities, revenue shares and aggregate productivities) is what determines the efficient inter-sectoral allocation. Notice that to find $\widetilde{Z}_{ls}$ it is necessary to solve for $\tilde{P}_s$, which implies to find firm's output prices in the efficient allocation. These prices can be obtained by solving the non-linear system that includes all firm-level prices, through numerical optimization. Once $\widetilde{Z}_{ls}$ are obtained, it is simple to calculate both gains from removing misallocation, using the counterfactual aggregate output generated by $\widetilde{A}_s$ and Z_{ls}.

Figure 3 – Sensitivity to elasticity of substitution across sectors



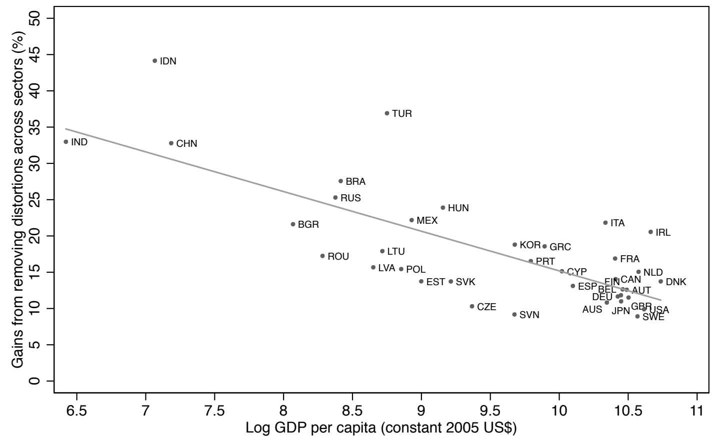
3 Inter-industry misallocation and development

Another important question about the relevance of inter-industry misallocation is whether its associated TFP loss is larger in less developed economies, as is the case with intra-industry misallocation, the core result of HK's paper. If the inter-sectoral gains vary systematically across countries, omitting the inter-sectoral component implies an under-estimation of the TFP gap attributed to factor misallocation, if the latter is computed only with intra-industry reforms, as in HK. In the case of the CD aggregator across sectors, the closed form solution for the TFP gains of removing inter-industry misallocation only requires information at the industry level. Thus, I use information from the socio-economic accounts of the World Input Output Database - WIOD (Timmer et al. (2015)), which contains industry-level data for 40 countries and 35 industries mostly at the 2-digit ISIC level, covering the overall economy, to compute those gains.

Figure 4 presents how the gains from inter-sectoral reallocation vary with the GDP per capita by country.[5] For this calculation, I use a gross output specification for the sectoral production function with 3 inputs (hours worked, capital and materials) and US cost shares. The linear correlation between both variables in this baseline is -0.75 (Figure 4 also shows the best linear fit). The negative correlation is robust to the use of value added specification or own country's cost shares in the production function; to restrict the set of sectors to only manufacturing industries and to measure labor with the wage bill and materials in nominal values to control for heterogeneity in labor and for differences in quality of intermediate inputs respectively, graphs shown in Figure 5. Therefore, there is evidence that less developed economies tend to have greater inter-sectoral gains for removing distortions. This is consistent with the insights of multi-country studies as Tombe (2015) or Święcki (2017) which focus on inter-sectoral misallocation, that find larger intersectoral distortions in poor countries. Thus, omitting the inter-sectoral component of the total gains from removing distortions understates the common TFP gaps attributed to firm-level misallocation.

[5] Each dot corresponds to the average value between 1995 and 2007 of the intersectoral gains calculated using (5) for each country and the average GDP per capita in constant 2005 US dollars obtained from the World Bank. The results are very similar if median values are used. Two small countries with many zeros in sectoral data were dropped from the WIOD sample (Luxembourg and Malta). Likewise, Taiwan was dropped to make comparable WIOD and World Bank data.

Figure 4 – TFP gains from removing inter-industry misallocation and GDP per capita



Note: Each dot corresponds to the average gains from removing inter-industry misallocation and the corresponding average GDP per capita in the period 1991-2007. The source of the data is WIOD and the World Bank development indicators.

If the inter-sectoral gains vary systematically across countries, omitting the inter-sectoral component implies an under-estimation of the TFP gap attributed to factor misallocation, if the latter is computed only with intra-industry reforms, as in HK. Since the inter-industry gains could be calculated with sectoral data, I use information from the socio-economic accounts of the World Input Output Database - WIOD (Timmer et al., 2015), which contains industry-level data for 40 countries and 35 industries mostly at the 2-digit ISIC level, covering the overall economy, to compute this dimension. Figure 4 presents how the gains from inter-sectoral reallocation vary with the GDP per capita by country[6]. For this calculation, I use a gross output specification for the sectoral production function with 3 inputs (hours worked, capital and materials) and US cost shares. The linear correlation between both variables in this baseline is -0.75 (Figure 4 also shows the best linear fit). The negative correlation is robust to the use of value added specification or own country's cost shares in the production function; to restrict the set of sectors to only manufacturing industries and to measure labor with the wage bill and materials in nominal values to control for heterogeneity in labor and for differences in quality of intermediate inputs respectively, graphs shown in Figure 5 below. Therefore, there is evidence that less developed economies tend to have greater inter-sectoral gains for removing distortions. This is consistent with the insights of multi-country studies as Tombe (2015) or Święcki (2017) which focus on inter-sectoral misallocation, that find larger intersectoral distortions in poor countries. Thus, omitting the inter-sectoral component of the total gains from removing distortions understates the common TFP gaps attributed to firm-level misallocation.

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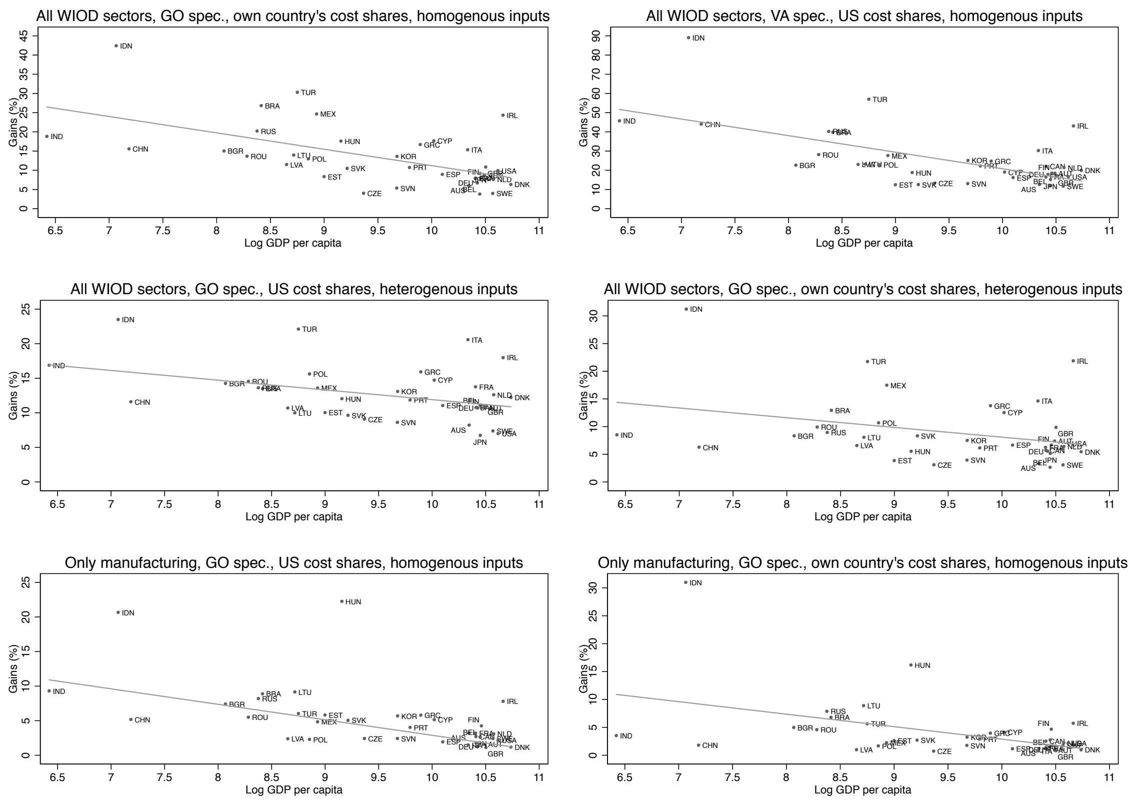
Figure 5 – Inter-sectoral gains and GDP per capita: Alternative specifications



Note: Averages 1994-2007. Data source: WIOD (Timmer et al., 2015), World Bank Development Indicators

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