A Appendix. Proofs of theoretical propositions

A.1 Proof of Proposition 1

The productivity cut-off for exporting $1/a_{ijs}$ is given by the condition

$$r_{ijs}\left(a_{ijs}\right)=\left(\frac{\tau_{ij}c_{js}a_{ijs}}{\alpha P_{is}}\right)^{1-\varepsilon}\theta_s Y_i=\varepsilon\left\{\left(1-d_s+\frac{d_s}{\lambda_j}\right)c_{js}f_{ij}-\frac{1-\lambda_j}{\lambda_j}t_s c_{js}f_{ej}\right\}.$$

Let $RHS=\left(1-d_s+\frac{d_s}{\lambda_j}\right)c_{js}f_{ij}-\frac{1-\lambda_j}{\lambda_j}t_s c_{js}f_{ej}$. Taking first derivatives, $\frac{\partial RHS}{\partial d_s}=\left(\frac{1}{\lambda_j}-1\right)c_{js}f_{ij}>0$ and $\frac{\partial RHS}{\partial t_s}=-\left(\frac{1}{\lambda_j}-1\right)c_{js}f_{ej}<0$ since $\lambda_j\epsilon\left(0,1\right)$. Also taking first derivatives, $\frac{\partial RHS}{\partial \lambda_j}=\frac{1}{\lambda_j^2}\left(t_s c_{js}f_{ej}-d_s c_{js}f_{ij}\right)<0$ if the loan exceeds the collateral as assumed, $t_s c_{js}f_{ej}<d_s c_{js}f_{ij}$. Taking second derivatives, $\frac{\partial^2 RHS}{\partial\lambda_j\partial d_s}=-\frac{1}{\lambda_j^2}c_{js}f_{ij}<0$ and $\frac{\partial^2 RHS}{\partial\lambda_j\partial t_s}=\frac{1}{\lambda_j^2}c_{js}f_{ej}>0$. Since revenues $r_{ijs}\left(a_{ijs}\right)$ are strictly increasing in productivity $1/a_{ijs}$, this proves Proposition 1.

A.2 Proof of Proposition 4

If firms sell to multiple destinations, they require outside capital for a fraction d_s of the fixed costs associated with entering each market. Companies then choose the optimal number of importers I, the price and quantity in each market to maximize worldwide export profits by solving

$$\max_{p,q,I,F}\pi_{js}\left(a\right)=\sum_{i=1}^{I}p_{ijs}\left(a\right)q_{ijs}\left(a\right)-\sum_{i=1}^{I}q_{ijs}\left(a\right)\tau_{ij}c_{js}a-\left(1-d_s\right)c_{js}\sum_{i=1}^{I}f_{ij}-\lambda_j F\left(a\right)-\left(1-\lambda_j\right)t_s c_{js}f_{ej} \quad (1)$$

subject to (1.1) $q_{ijs}\left(a\right)=\frac{p_{ijs}(a)^{-\varepsilon}\theta_s Y_i}{P_{is}^{1-\varepsilon}}$,

(1.2) $A_{js}\left(a\right)\equiv\sum_{i=1}^{I}p_{ijs}\left(a\right)q_{ijs}\left(a\right)-\sum_{i=1}^{I}q_{ijs}\left(a\right)\tau_{ij}c_{js}a-\left(1-d_s\right)c_{js}\sum_{i=1}^{I}f_{ij}\geq F\left(a\right)$, and

(1.3) $B_{js}\left(a\right)\equiv-d_s c_{js}\sum_{i=1}^{I}f_{ij}+\lambda_j F\left(a\right)+\left(1-\lambda_j\right)t_s c_{js}f_{ej}\geq 0.$

With competitive credit markets, investors break even in expectation and producers adjust the payment $F\left(a\right)$ so that $B_{js}\left(a\right)=0$. Firms have to use their limited collateral to fund their activities in multiple destinations. However, export revenues in one market are not directly affected by sales in a different market. This implies that all exporters add trade partners in the same decreasing order of profitability (determined by Y_i, P_{is}, τ_{ij}, and f_{ij}) until they exhaust their financial resources. If the liquidity constraint (1.2) does not bind, sellers therefore set their first-best price, quantity, revenues and profit levels in each market they choose to service.

For any given I, there is a productivity cut-off $1/a_{js,I}$ below which (1.2) binds. Plugging $B_{js}\left(a\right)=0$ and the optimal price and quantity in $A_{js}\left(a_{js,I}\right)=F\left(a_{js,I}\right)$, this cut-off is given by

$$\sum_{i=1}^{I}r_{ijs}\left(a_{js,I}\right)=\sum_{i=1}^{I}\left(\frac{\tau_{ij}c_{js}a_{js,I}}{\alpha P_{is}}\right)^{1-\varepsilon}\theta_s Y_i=\varepsilon\left\{\left(1-d_s+\frac{d_s}{\lambda_j}\right)c_{js}\sum_{i=1}^{I}f_{ij}-\frac{1-\lambda_j}{\lambda_j}t_s c_{js}f_{ej}\right\}.$$

The left-hand side of this expression is increasing in $1/a_{js,I}$, while the right-hand side inherits all properties of RHS above. This implies that $\frac{\partial\left(1/a_{js,I}\right)}{\partial\lambda_j}<0$, $\frac{\partial\left(1/a_{js,I}\right)}{\partial d_s}>0$, $\frac{\partial\left(1/a_{js,I}\right)}{\partial t_s}<0$, $\frac{\partial^2\left(1/a_{js,I}\right)}{\partial\lambda_j\partial d_s}<0$ and $\frac{\partial^2\left(1/a_{js,I}\right)}{\partial\lambda_j\partial t_s}>0$. Since all firms add foreign markets in the same order of profitability, country j exports to I destinations only if at least one firm in j is more productive than $1/a_{js,I}$ and exports to these I destinations. This proves Proposition 4.

A.3 Proof of Proposition 5

When firms need outside capital for a fraction d_s of both fixed and variable costs, their maximization problem becomes

$$\max_{p,q,F} \pi_{ijs}(a) = p_{ijs}(a) q_{ijs}(a) - (1-d_s) q_{ijs}(a) \tau_{ij} c_{js} a - (1-d_s) c_{js} f_{ij} - \lambda_j F(a) - (1-\lambda_j) t_s c_{js} f_{ej} \quad (2)$$

subject to (1.1) $q_{ijs}(a) = \frac{p_{ijs}(a)^{-\varepsilon} \theta_s Y_i}{P_{is}^{1-\varepsilon}}$,

(1.2) $A_{ijs}(a) \equiv p_{ijs}(a) q_{ijs}(a) - (1-d_s) q_{ijs}(a) \tau_{ij} c_{js} a - (1-d_s) c_{js} f_{ij} \geq F(a)$, and

(1.3) $B_{ijs}(a) \equiv -d_s q_{ijs}(a) \tau_{ij} c_{js} a - d_s c_{js} f_{ij} + \lambda_j F(a) + (1-\lambda_j) t_s c_{js} f_{ej} \geq 0$.

With competitive credit markets, investors break even in expectation and producers adjust the payment $F(a)$ so that $B_{ijs}(a) = 0$. If the liquidity constraint (1.2) does not bind, firms thus export at their first-best price, quantity, revenues and profit levels as in Melitz (2003). This will be the case for firms with productivity above $1/a_{ijs}^H$, defined by $A_{ijs}\left(a_{ijs}^H\right) = F\left(a_{ijs}^H\right)$, or

$$\left[1 - (1-d_s)\alpha - \frac{d_s \alpha}{\lambda_j}\right] \left(\frac{\tau_{ij} c_{js} a_{ijs}^H}{\alpha P_{is}}\right)^{1-\varepsilon} \theta_s Y_i = \left(1 - d_s + \frac{d_s}{\lambda_j}\right) c_{js} f_{ij} - \frac{1-\lambda_j}{\lambda_j} t_s c_{js} f_{ej}. \quad (3)$$

The right-hand side of this expression is exactly RHS above and exhibits the same properties. The left-hand side LHS is increasing in productivity $1/a_{ijs}^H$. Since it does not depend on t_s or the interaction term $t_s \lambda_j$, $\frac{\partial\left(1/a_{ijs}^H\right)}{\partial t_s} < 0$ and $\frac{\partial^2\left(1/a_{ijs}^H\right)}{\partial \lambda_j \partial t_s} > 0$ because $\frac{\partial RHS}{\partial t_s} < 0$ and $\frac{\partial^2 RHS}{\partial \lambda_j \partial t_s} > 0$. Taking first and second derivatives, $\frac{\partial LHS}{\partial d_s} = \left[\alpha - \frac{\alpha}{\lambda_j}\right] \left(\frac{\tau_{ij} c_{js} a_{ijs}^H}{\alpha P_{is}}\right)^{1-\varepsilon} \theta_s Y_i < 0$, $\frac{\partial LHS}{\partial \lambda_j} = \frac{d_s \alpha}{\lambda_j^2} \left(\frac{\tau_{ij} c_{js} a_{ijs}^H}{\alpha P_{is}}\right)^{1-\varepsilon} \theta_s Y_i > 0$ and $\frac{\partial^2 LHS}{\partial \lambda_j \partial d_s} = \frac{\alpha}{\lambda_j^2} \left(\frac{\tau_{ij} c_{js} a_{ijs}^H}{\alpha P_{is}}\right)^{1-\varepsilon} \theta_s Y_i > 0$ because $\lambda_j \epsilon (0,1)$. Since the signs of these three derivatives are opposite to those of $\frac{\partial RHS}{\partial d_s} > 0$, $\frac{\partial RHS}{\partial \lambda_j} < 0$ and $\frac{\partial^2 RHS}{\partial \lambda_j \partial d_s} < 0$, it follows that $\frac{\partial\left(1/a_{ijs}^H\right)}{\partial d_s} > 0$, $\frac{\partial\left(1/a_{ijs}^H\right)}{\partial \lambda_j} < 0$ and $\frac{\partial^2\left(1/a_{ijs}^H\right)}{\partial \lambda_j \partial d_s} < 0$. The comparative statics for $1/a_{ijs}^H$ are thus identical to those for $1/a_{ijs}$ above.

When firms finance only fixed costs externally as in Section 3.3, maximizing profits is equivalent to maximizing net revenues $A_{ijs}(a)$. First-best prices then also maximize firms' possible payment to the investor $F(a)$ and hence the probability of exporting. In contrast, when firms require external capital for both fixed and variable costs, firms with productivity below $1/a_{ijs}^H$ have an incentive to reduce their export scale from the unconstrained optimum. This occurs because exporting larger quantities requires more outside finance, which increases the repayment $F(a)$ necessary to meet the investor's participation constraint. Given (1.1), constrained firms thus sell lower quantities at higher prices. Because deviating from the first-best lowers profits, they scale down (and increase the price) as little as possible to ensure that investors can break even. Plugging (1.1) and $B_{ijs}(a) = 0$ into (1.2) and setting $A_{ijs}(a) = F(a)$, firms' prices solve

$$\frac{p_{ijs}(a)^{1-\varepsilon} \theta_s Y_i}{P_{is}^{1-\varepsilon}} - \left(1 - d_s + \frac{d_s}{\lambda_j}\right) \tau_{ij} c_{js} a \frac{p_{ijs}(a)^{-\varepsilon} \theta_s Y_i}{P_{is}^{1-\varepsilon}} = \left(1 - d_s + \frac{d_s}{\lambda_j}\right) c_{js} f_{ij} - \frac{1-\lambda_j}{\lambda_j} t_s c_{js} f_{ej}. \quad (4)$$

Constrained firms choose a price between the first best $\frac{\tau_{ij} c_{js} a}{\alpha}$ and the price that maximizes the left-hand side of (4) LHS. In this range, LHS is increasing in $p_{ijs}(a)$. To see this, take the first derivative $\frac{\partial LHS}{\partial p_{ijs}} = \frac{p_{ijs}(a)^{-\varepsilon-1} \theta_s Y_i}{P_{is}^{1-\varepsilon}} \cdot \left[(1-\varepsilon) p_{ijs}(a) + \varepsilon \left(1 - d_s + \frac{d_s}{\lambda_j}\right) \tau_{ij} c_{js} a\right]$. Since $p_{ijs}(a) \geq \frac{\tau_{ij} c_{js} a}{\alpha}$, $\frac{\partial LHS}{\partial p_{ijs}} \geq \frac{p_{ijs}(a)^{-\varepsilon-1} \theta_s Y_i}{P_{is}^{1-\varepsilon}} \cdot \varepsilon \tau_{ij} c_{js} a \left(-d_s + \frac{d_s}{\lambda_j}\right) > 0$ because $\lambda_j \epsilon (0,1)$.

The right-hand side of (4) is the same RHS as above. Since LHS does not depend on t_s or $t_s\lambda_j$, $\frac{\partial p_{ijs}}{\partial t_s} < 0$ and $\frac{\partial^2 p_{ijs}}{\partial t_s \partial \lambda_j} > 0$ because $\frac{\partial RHS}{\partial t_s} < 0$ and $\frac{\partial^2 RHS}{\partial \lambda_j \partial t_s} > 0$. Taking first and second derivatives, $\frac{\partial LHS}{\partial d_s} = \left(1 - \frac{1}{\lambda_j}\right) \tau_{ij} c_{js} a \frac{p_{ijs}(a)^{-\varepsilon} \theta_s Y_i}{P_{is}^{1-\varepsilon}} < 0$, $\frac{\partial LHS}{\partial \lambda_j} = \frac{d_s}{\lambda_j^2} \tau_{ij} c_{js} a \frac{p_{ijs}(a)^{-\varepsilon} \theta_s Y_i}{P_{is}^{1-\varepsilon}} > 0$ and $\frac{\partial^2 LHS}{\partial \lambda_j \partial d_s} = \frac{1}{\lambda_j^2} \tau_{ij} c_{js} a \frac{p_{ijs}(a)^{-\varepsilon} \theta_s Y_i}{P_{is}^{1-\varepsilon}} > 0$ because $\lambda_j \epsilon (0,1)$. Since the signs of these three derivatives are opposite to those of $\frac{\partial RHS}{\partial d_s} > 0$, $\frac{\partial RHS}{\partial \lambda_j} < 0$ and $\frac{\partial^2 RHS}{\partial \lambda_j \partial d_s} < 0$, it follows that $\frac{\partial p_{ijs}}{\partial d_s} > 0$, $\frac{\partial p_{ijs}}{\partial \lambda_j} < 0$ and $\frac{\partial^2 p_{ijs}}{\partial \lambda_j \partial d_s} < 0$. Since export quantities and revenues are decreasing in the price, the comparative statics for them are reversed: $\frac{\partial r_{ijs}}{\partial \lambda_j} > 0$, $\frac{\partial r_{ijs}}{\partial d_s} < 0$, $\frac{\partial r_{ijs}}{\partial t_s} > 0$, $\frac{\partial^2 r_{ijs}}{\partial \lambda_j \partial d_s} > 0$ and $\frac{\partial^2 r_{ijs}}{\partial \lambda_j \partial t_s} < 0$. This proves Proposition 5.

Some potentially profitable exporters will not be able to sell abroad. The left-hand side of (4) is maximized at $p_{ijs}^L(a) = \left(1 - d_s + \frac{d_s}{\lambda_j}\right) \frac{\tau_{ij} c_{js} a}{\alpha}$ and firms have no incentive to raise their price above this level. Therefore, firms with productivity below $1/a_{ijs}^L$ cannot export because, even if they set this price and give all revenues to the investor in case of repayment, the investor would not break even. Plugging $p_{ijs}^L(a)$ into (4), this cut-off is defined by

$$\left(1 - d_s + \frac{d_s}{\lambda_j}\right)^{1-\varepsilon} \left(\frac{\tau_{ij} c_{js} a_{ijs}^L}{\alpha P_{is}}\right)^{1-\varepsilon} \theta_s Y_i = \varepsilon \left\{ \left(1 - d_s + \frac{d_s}{\lambda_j}\right) c_{js} f_{ij} - \frac{1-\lambda_j}{\lambda_j} t_s c_{js} f_{ej} \right\}. \quad (5)$$

The right-hand side of (5) inherits the properties of RHS above. The left-hand side of (5) LHS is increasing in productivity $1/a_{ijs}^L$. Since it does not depend on t_s or $t_s\lambda_j$, $\frac{\partial\left(1/a_{ijs}^L\right)}{\partial t_s} < 0$ and $\frac{\partial^2\left(1/a_{ijs}^L\right)}{\partial \lambda_j \partial t_s} > 0$ because $\frac{\partial RHS}{\partial t_s} < 0$ and $\frac{\partial^2 RHS}{\partial \lambda_j \partial t_s} > 0$. Taking first derivatives, $\frac{\partial LHS}{\partial d_s} = (\varepsilon - 1)\left(1 - d_s + \frac{d_s}{\lambda_j}\right)^{-\varepsilon} \left(1 - \frac{1}{\lambda_j}\right) \left(\frac{\tau_{ij} c_{js} a_{ijs}^L}{\alpha P_{is}}\right)^{1-\varepsilon} \theta_s Y_i < 0$ and $\frac{\partial LHS}{\partial \lambda_j} = (\varepsilon - 1)\left(1 - d_s + \frac{d_s}{\lambda_j}\right)^{-\varepsilon} \frac{d_s}{\lambda_j^2} \left(\frac{\tau_{ij} c_{js} a_{ijs}^L}{\alpha P_{is}}\right)^{1-\varepsilon} \theta_s Y_i > 0$ because $\lambda_j \epsilon (0,1)$. Since the signs of these derivatives are opposite to those of $\frac{\partial RHS}{\partial d_s} > 0$ and $\frac{\partial RHS}{\partial \lambda_j} < 0$, it follows that $\frac{\partial\left(1/a_{ijs}^L\right)}{\partial d_s} > 0$ and $\frac{\partial\left(1/a_{ijs}^L\right)}{\partial \lambda_j} < 0$. While $\frac{\partial^2 RHS}{\partial \lambda_j \partial d_s} < 0$ can be signed, however, the sign of $\frac{\partial^2 LHS}{\partial \lambda_j \partial d_s} = (\varepsilon - 1) \left(\frac{\tau_{ij} c_{js} a_{ijs}^L}{\alpha P_{is}}\right)^{1-\varepsilon} \theta_s Y_i \cdot \left(1 - d_s + \frac{d_s}{\lambda_j}\right)^{-\varepsilon-1} \frac{1}{\lambda_j^2} \left[1 + d_s(\varepsilon - 1)\left(1 - \frac{1}{\lambda_j}\right)\right] \gtreqless 0$ is ambiguous because $\lambda_j \epsilon (0,1)$. Intuitively, more productive firms have higher revenues to offer in case of repayment, but they also require more external capital for their variable costs since they operate at a larger scale. The former effect dominates and $\frac{\partial^2 LHS}{\partial \lambda_j \partial d_s} > 0$ whenever $1 + d_s(\varepsilon - 1)\left(1 - \frac{1}{\lambda_j}\right) > 0$. In that case we can unambiguously conclude that $\frac{\partial^2\left(1/a_{ijs}^L\right)}{\partial \lambda_j \partial d_s} < 0$. This is a sufficient but not a necessary condition: $\frac{\partial^2\left(1/a_{ijs}^L\right)}{\partial \lambda_j \partial d_s} < 0$ will hold even if $\frac{\partial^2 LHS}{\partial \lambda_j \partial d_s} < 0$ as long as RHS falls faster with $d_s\lambda_j$ than LHS.[1] Given results in the corporate finance literature that larger, more productive firms are less likely to be credit constrained, as well as my own empirical findings, I assume that $\frac{\partial^2\left(1/a_{ijs}^L\right)}{\partial \lambda_j \partial d_s} < 0$ is satisfied. This confirms that Proposition 1 holds when firms borrow for both fixed and variable costs.

[1] The necessary condition is $\frac{\varepsilon-1}{\lambda_j^2}\left(1 - d_s + \frac{d_s}{\lambda_j}\right)^{-\varepsilon-1} \left[1 + d_s(\varepsilon - 1)\left(1 - \frac{1}{\lambda_j}\right)\right] \left(\frac{\tau_{ij} c_{js}}{\alpha P_{is}}\right)^{1-\varepsilon} \theta_s Y_i > -\frac{\varepsilon}{\lambda_j^2} c_{js} f_{ij}$.

A.4 Proof of Proposition 6

Aggregating across firms, total exports from country j to country i in sector s are $M_{ijs} = \left(\frac{\tau_{ij}c_{js}}{\alpha P_{is}}\right)^{1-\varepsilon}\theta_s Y_i N_{js}\left[\int_{a_L}^{a_{ijs}^H} a^{1-\varepsilon}dG\left(a\right) + \int_{a_{ijs}^H}^{a_{ijs}^L}\beta_{ijs}(a)a^{1-\varepsilon}dG\left(a\right)\right]$, where N_{js} is the exogenous measure of active producers. The first term in the brackets corresponds to companies trading at first-best levels. The second term captures the reduced revenues of constrained exporters, which for simplicity have been expressed as a fraction $\beta_{ijs}(a)\epsilon\left(0,1\right)$ of first-best revenues. Taking first derivatives, $\frac{\partial M_{ijs}}{\partial a_{ijs}^H} = \left(\frac{\tau_{ij}c_{js}}{\alpha P_{is}}\right)^{1-\varepsilon}\theta_s Y_i N_{js}\left(a_{ijs}^H\right)^{1-\varepsilon}\left[1-\beta_{ijs}(a_{ijs}^H)\right] = 0$ since $\beta_{ijs}(a_{ijs}^H) = 1$, $\frac{\partial M_{ijs}}{\partial a_{ijs}^L} = \left(\frac{\tau_{ij}c_{js}}{\alpha P_{is}}\right)^{1-\varepsilon}\theta_s Y_i N_{js}\beta_{ijs}(a_{ijs}^L)\left(a_{ijs}^L\right)^{1-\varepsilon} > 0$, and $\frac{\partial M_{ijs}}{\partial \beta_{ijs}(a)} = \left(\frac{\tau_{ij}c_{js}}{\alpha P_{is}}\right)^{1-\varepsilon}\theta_s Y_i N_{js}a^{1-\varepsilon} > 0$. Given the comparative statics for $1/a_{ijs}^H$, $1/a_{ijs}^L$ and $\beta_{ijs}(a)$ above, this implies that $\frac{\partial M_{ijs}}{\partial \lambda_j} > 0$, $\frac{\partial M_{ijs}}{\partial d_s} < 0$, $\frac{\partial M_{ijs}}{\partial t_s} > 0$, $\frac{\partial^2 M_{ijs}}{\partial \lambda_j \partial d_s} > 0$ and $\frac{\partial^2 M_{ijs}}{\partial \lambda_j \partial t_s} < 0$. This proves Proposition 6.

B Appendix. Data sources

GDP and GDP per capita: from the *Penn World Tables 6.1.*

Corruption and rule of law: from La Porta et al. (1998).

Physical and human capital endowments per capita: from Caselli (2005). The stock of physical capital is obtained according to the perpetual inventory method as $K_t = I_t + \delta K_{t-1}$, where I_t is investment and δ is the depreciation rate. The initial capital stock K_o is computed as $I_0/\left(g+\delta\right)$, where I_0 is the earliest value of investment available, and g is the average geometric growth rate of investment before 1970. Human capital per worker is calculated from the average years of schooling in a country with Mincerian non-linear returns to education. It is measured as $h = e\varphi\left(s\right)$, where s is the average years of schooling in the population over 25 years old, and $\varphi\left(s\right)$ is piecewise linear with slope 0.13 for $s \leq 4$, 0.10 for $4 < s \leq 8$, and 0.07 for $8 < s$.

Natural resources per worker: from the World Bank's *Expanding the Measure of Wealth.*

Sectors' factor intensity: from Braun (2003).

Output and number of establishments by sector: from *UNIDO.*

Consumer price index: from the IMF's *International Financial Statistics.*