El modelo de Melitz (2003)

Firmas y Comercio Internacional

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Melitz (2003)

- Melitz (2003) develops a model that can successfully explain many of the firm-level features seen in the previous slides
- Main features:
  - Firms are heterogeneous in productivity
  - Fixed costs of exporting
- Main implications:
  - Only the most productive firms export
  - Trade liberalization reallocates market shares towards most productive firms
  - This reallocation works like an increase in industry's productivity

Demand

- Representative consumer has CES preferences

$$U = \left[\int_{\omega \in \Omega} q\left(\omega\right)^{\frac{\sigma-1}{\sigma}} \, d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

where Ω is the set of available varieties

- Consumers maximize U subject to

$$\int_{\omega} p\left(\omega\right) q\left(\omega\right) d\omega = R$$

- This yields demand for individual variety ω:

$$q\left(\omega\right) = \left[\frac{p\left(\omega\right)}{P} \right]^{-\sigma} \frac{R}{P}$$

where P is the CES price index

$$P = \left[\int_{\omega} p\left(\omega\right)^{1-\sigma} \, d\omega \right]^{\frac{1}{1-\sigma}}$$

Production

- There is a continuum of firms each producing a different variety ω
- One factor of production, labor, inelastically supplied at its aggregate level L
- There are increasing returns to scale in production:

$$l = f + \frac{q}{\varphi}$$

All firms share the same fixed cost of production f but have different productivity levels indexed by $\varphi > 0$

- Each firm's constant marginal cost is given by

$$MC(\varphi) = \frac{w}{\varphi}$$

where w is the wage from now normalized to one

Production

- All firms face a residual demand curve with elasticity σ
- All firms set the same markup over marginal cost

$$p(\varphi) = \frac{\sigma}{\sigma - 1}\frac{1}{\varphi}$$

- Firm revenue and profit are determined by φ and aggregate variables:

$$r(\varphi) = R\left(P\frac{\sigma - 1}{\sigma}\varphi\right)^{\sigma - 1}$$
$$\pi(\varphi) = \frac{1}{\sigma}r(\varphi) - f$$

Production

- Note that more productive firms have higher output and higher revenues

$$\frac{q(\varphi_1)}{q(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma} \text{ and } \frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$$

- Variable profits (gross of fixed costs) are proportional to revenues for all firms (hence also increase in φ)
- Higher φ implies higher revenue productivity, which is typically the measured firm-level productivity

$$\frac{r(\varphi)}{l(\varphi)} = \frac{\sigma}{\sigma-1}\left[1 - \frac{f}{l(\varphi)}\right]$$

  - Crucial to take fixed costs into account - revenue per variable input is independent of φ
- The model could be reinterpreted as φ representing differences in quality rather than in costs

Aggregation

- An equilibrium will be characterized by:
  - Mass M of firms
  - Distribution $\mu(\varphi)$ of productivity levels
- Since all firms with productivity φ charge the same price $p(\varphi)$ the price index can be written as

$$P = \left[\int_0^\infty p(\varphi)^{1-\sigma} M\mu(\varphi)\, d\varphi\right]^{\frac{1}{1-\sigma}}$$

- Given $\mu(\varphi)$ define a weighted average of φ as

$$\widetilde{\varphi} \equiv \left(E\left[\varphi^{\sigma-1}\right]\right)^{\frac{1}{\sigma-1}} = \left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)\, d\varphi\right]^{\frac{1}{\sigma-1}}$$

Aggregation

- $\widetilde{\varphi}$ summarizes all the information in $\mu(\varphi)$ relevant for aggregate variables:

$$P = M^{1/(1-\sigma)} p(\widetilde{\varphi}) \qquad R = PQ = Mr(\widetilde{\varphi})$$
$$Q = M^{\sigma/(\sigma-1)} q(\widetilde{\varphi}) \qquad \Pi = M\pi(\widetilde{\varphi})$$

- $\widetilde{\varphi}$ represents aggregate productivity

Entry and Exit

Assumptions

- Firms are identical prior to entry and must pay a fixed investment cost f_e to enter
- Upon entry firms draw a productivity level φ from a common distribution $g(\varphi)$
- After observing their productivity firms decide whether to exit or to remain active
- Firms remaining active face a constant probability δ of a bad shock that would force them to exit

Entry and Exit

Implications

- In a stationary equilibrium, a firm either exits immediately or produces and earns the same profit $\pi(\varphi)$ each period
- Given a realization of φ, expected value of a firm (no time discounting) is

$$v(\varphi) = \max\left\{0, \sum_{t=0}^{\infty}(1-\delta)^t\,\pi(\varphi)\right\} = \max\left\{0, \frac{\pi(\varphi)}{\delta}\right\}$$

- There exists a unique productivity cutoff φ^* such that firms with $\varphi \geq \varphi^*$ produce and firms with $\varphi < \varphi^*$ exit

$$\pi(\varphi^*) = 0$$

Entry and Exit

- Distribution of active firms $\mu(\varphi)$ will be given by the conditional of $g(\varphi)$ on $[\varphi^*, \infty)$

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*)} & \text{if } \varphi \geq \varphi^* \\ 0 & \text{otherwise} \end{cases}$$

- This defines the aggregate productivity $\widetilde{\varphi}$ as a function of the cutoff φ^*:

$$\widetilde{\varphi}(\varphi^*) = \left[\frac{1}{1 - G(\varphi^*)} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g(\varphi)\, d\varphi\right]^{\frac{1}{\sigma-1}}$$

Free Entry Condition

- Let $\overline{\pi} = \Pi / M$ denote the average profits per period across all active firms
- Free entry requires that the expected profits are equal to the fixed cost of entry:

$$0 \times G(\varphi^*) + \frac{\overline{\pi}}{\delta} \times [1 - G(\varphi^*)] = f_e$$

- **Free Entry condition (FE)**:

$$\overline{\pi} = \frac{\delta f_e}{1 - G(\varphi^*)}$$

- If firms are less likely to survive (higher φ^*), they need to be compensated with higher average profits (higher $\overline{\pi}$)

Zero Cutoff Profit Condition

- Definition of φ^* can be manipulated to yield another relationship between φ^* and $\overline{\pi}$

$$\pi(\varphi^*) = 0 \Leftrightarrow r(\varphi^*) = \sigma f$$

$$\Leftrightarrow \overline{r} = r(\tilde{\varphi}) = \left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} r(\varphi^*) = \left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1} \sigma f$$

$$\overline{\pi} = \pi(\tilde{\varphi}) = \frac{r(\tilde{\varphi})}{\sigma} - f = f\left[\left(\frac{\widetilde{\varphi}}{\varphi^*}\right)^{\sigma-1} - 1\right]$$

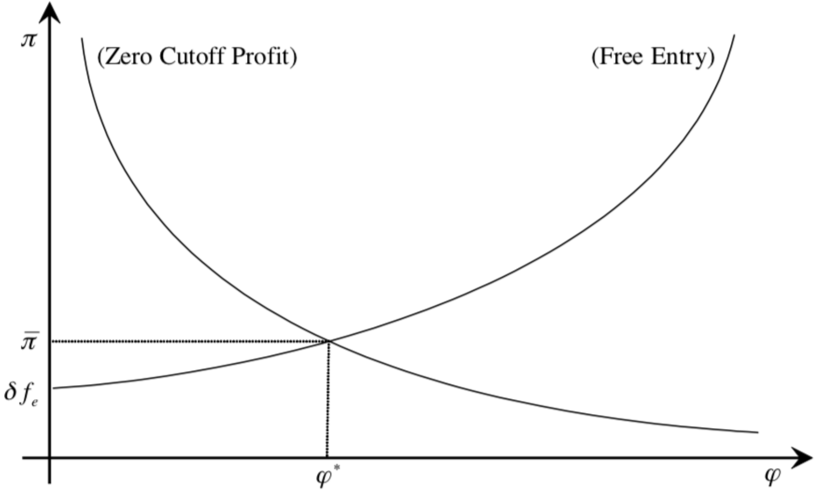
- The last expression is the **Zero Cutoff Profit condition (ZCP):**

$$\overline{\pi} = f\left[\left(\frac{\widetilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right]$$

Zero Cutoff Profit Condition

- As $\varphi^* \uparrow$ two effects on average profits:
  - $\overline{\pi} \uparrow$ because there are more productive firms on average in the market and they have higher profits
  - $\overline{\pi} \downarrow$ because other firms are more productive, there is more competition (lower price index)
- Whether ZCP is upward or downward sloping depends on the distribution of firms:
  - If right tail is thick enough (lots of very productive firms) then downward sloping
    - ★ True for commonly used distributions
  - For a special case of Pareto distribution the ZCP is flat because $\tilde{\varphi}/\varphi^*$ is constant

Autarky Equilibrium



Autarky Equilibrium

- FE and ZCP conditions uniquely determine $\overline{\pi}$ and φ^*
- Last endogenous variable to be determined is the measure of firms M
  - $L =$ total expenditure $=$total revenues $=R$
  - $R = M\overline{r}$
  - $\overline{r} = \sigma\left(\overline{\pi} + f\right)$
  - $M = \frac{L}{\sigma(\overline{\pi}+f)}$

Trade

- Without trade costs all active firms export and industry productivity is not affected by trade ($\tilde{\varphi}$ fixed)
- With only variable trade costs would still get counterfactual prediction that all firms exports
- To achieve self-selection into exporting the model needs fixed costs of exporting
  - In order to export firm needs to pay an additional fixed cost f_x after learning its productivity φ
- Include standard iceberg costs:
  - Need to send τ units for one unit to arrive
- Consider a world with $n + 1$ symmetric countries (n is $\#$ countries different to home)
  - Asymmetric case difficult to handle analytically in the full generality of the model

Production

- Firm with productivity φ
  - Sets price $p = \frac{\sigma}{\sigma-1}\frac{1}{\varphi}$ on domestic market
  - Earn revenues $r_d(\varphi) = R\left(P\varphi\frac{\sigma-1}{\sigma}\right)^{\sigma-1}$ from domestic sales
- If the firm chooses to export to a particular market
  - Sets export price $p_x = \tau\frac{\sigma}{\sigma-1}\frac{1}{\varphi}$
  - Earn export revenues $r_x(\varphi) = \tau^{1-\sigma}R_x\left(P_x\varphi\frac{\sigma-1}{\sigma}\right)^{\sigma-1}$
- Given symmetry $P = P_x$, $R = R_x$
  - If a firm exports, it exports to all countries
- Then:

$$\pi_d(\varphi) = \frac{r_d(\varphi)}{\sigma} - f, \quad \pi_x(\varphi) = \frac{r_x(\varphi)}{\sigma} - f_x$$

Cutoffs

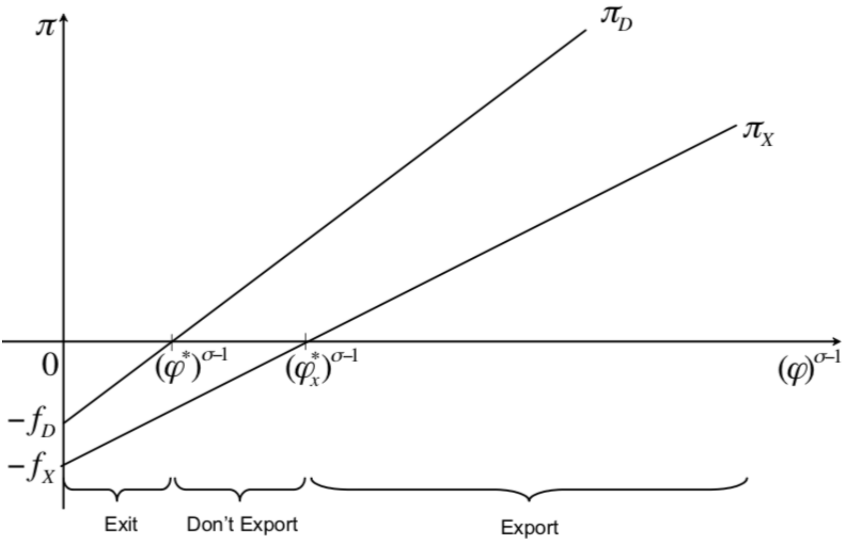
- Now we need to find:
  - Domestic cutoff φ^*
  - Exporting cutoff φ_x^*
- Exporting cutoff φ_x^* is such that $\pi_x(\varphi_x^*) = 0$

$$\begin{aligned}
\frac{\tau^{1-\sigma} r_d(\varphi_x^*)}{\sigma} - f_x &= 0 \\
\frac{\tau^{1-\sigma} r_d(\varphi^*)}{\sigma} \left(\frac{\varphi_x^*}{\varphi^*}\right)^{\sigma-1} - f_x &= 0 \\
\tau^{1-\sigma} f \left(\frac{\varphi_x^*}{\varphi^*}\right)^{\sigma-1} - f_x &= 0 \\
\varphi_x^* &= \varphi^* \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}
\end{aligned}$$

so we just need to find φ^*

- Assume that $\tau^{\sigma-1} f_x > f$ so that $\varphi_x^* > \varphi^*$

Selection into Exports



Cutoffs

- We need to find $(\text{ZCP})^{\text{T}}$ and FE under trade
- Define:
  - $\widetilde{\varphi}(\varphi^*)$ average productivity of producing firms
  - $\widetilde{\varphi}_x(\varphi_x^*)$ average productivity of exporting firms
- Average profits now depend on domestic and export profits:

$$\overline{\pi} = \pi_d(\widetilde{\varphi}) + p_x n \pi_x(\widetilde{\varphi}_x)$$

where p_x is the probability of exporting $= \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}$

Equilibrium Conditions

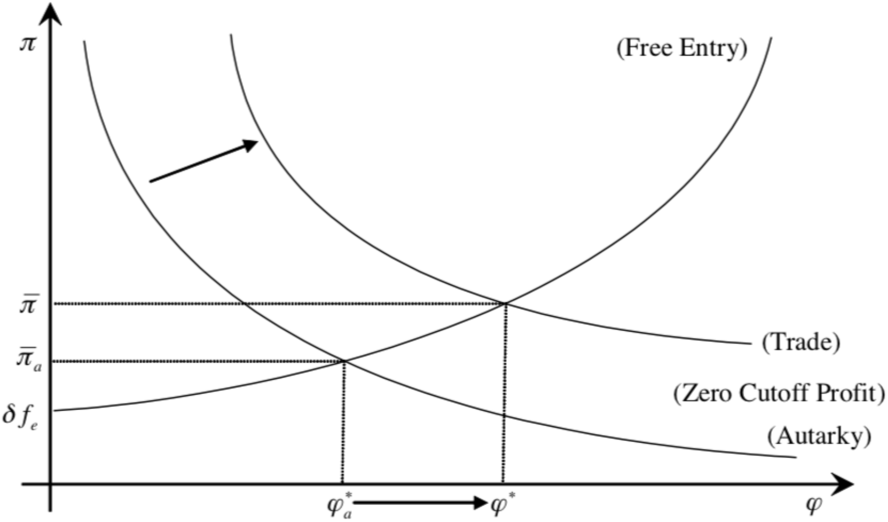
- Zero cutoff profit condition under trade $(\text{ZCP})^{\text{T}}$ is:

$$\overline{\pi} = f\left[\left(\frac{\widetilde{\varphi}\left(\varphi^{*}\right)}{\varphi^{*}}\right)^{\sigma-1} - 1\right] + p_x n f_x \left[\left(\frac{\widetilde{\varphi}_x\left(\varphi^{*}\right)}{\varphi_x^{*}\left(\varphi^{*}\right)}\right)^{\sigma-1} - 1\right]$$

- Free-entry condition is the same:

$$\overline{\pi} = \frac{\delta f_e}{1 - G\left(\varphi^{*}\right)}$$

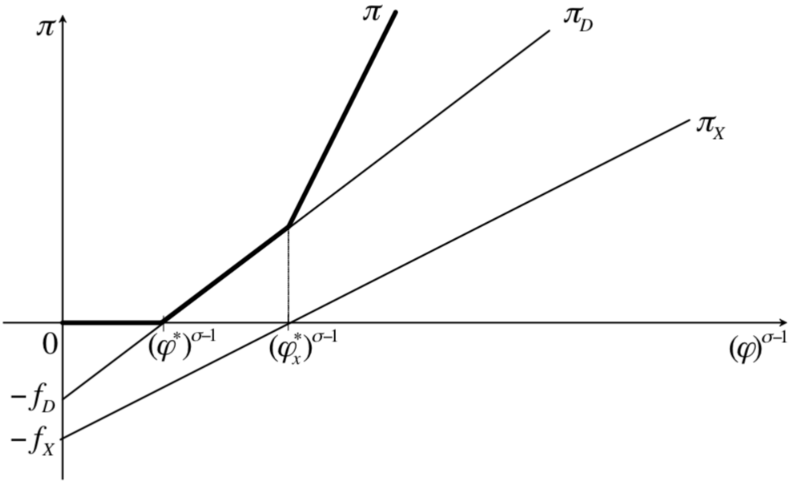
Impact of Trade



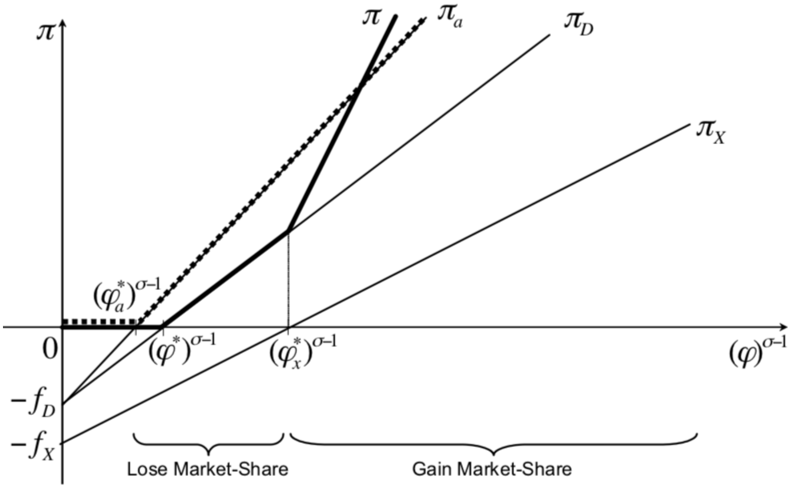
Impact of Trade

- Trade causes some of the least productive firms to exit: $\varphi^* > \varphi_a^*$ (where φ_a^* is the production cutoff under autarky)
  - Demand faced by firms that can export $\uparrow$
  - Demand for labor by these firms $\uparrow$
  - Real wage $\uparrow$
  - Less productive firms cannot afford to pay wage and exit
- In domestic market every firm's profits $\downarrow$ because of entry of productive exporters from abroad
- Exporters: only most productive gain overall
  - gain in export market
  - lose in domestic market

Impact of Trade



Impact of Trade



Impact of Trade (I)

- Measure of domestic firms decreases but the overall product variety rises:
  - Number of domestic firms can be computed again as:

$$L = M\bar{r} \Rightarrow L = M\left(r_d\left(\tilde{\varphi}\right) + p_x n r_x\left(\tilde{\varphi}_x\right)\right)$$
$$M = \frac{L}{\sigma\left(\pi_d\left(\widetilde{\varphi}\right) + f + p_x n \pi_x\left(\widetilde{\varphi}_x\right) + p_x n f_x\right)}$$
$$M = \frac{L}{\sigma\left(\overline{\pi} + f + p_x n f_x\right)}$$

  - Number of varieties available for consumers is simply: $M_t = M + M_x = M + p_x M = \left(1 + p_x\right) M$

Impact of Trade (II)

- Aggregate productivity increases
  - Total average productivity can be measured as:
    $\widetilde{\varphi}_t = \left[\frac{1}{M_t} \left(M \widetilde{\varphi}^{\sigma-1} + M_x \left(\frac{\widetilde{\varphi}_x}{\tau} \right)^{\sigma-1} \right) \right]^{\frac{1}{\sigma-1}}$
  - In terms of aggregation, $\widetilde{\varphi}_t$ satisfies the same properties of $\widetilde{\varphi}_a$ under autarky, so the price index can be rewritten as:

$$P = M_t^{1/(1-\sigma)} \frac{\sigma}{\sigma-1} \left(\frac{1}{\widetilde{\varphi}_t} \right)$$

Impact of Trade (III)

- Welfare (W) unambiguously rises
  - Measure of welfare is real wage: $W = 1/P = M_t^{1/(\sigma-1)} \frac{\sigma-1}{\sigma} (\widetilde{\varphi}_t)$
  - The gains from trade can be computed as: $\frac{W}{W_a} - 1$ where:

$$\frac{W}{W_a} = \left(\frac{M_t}{M_a}\right)^{1/(\sigma-1)} \frac{(\widetilde{\varphi}_t)}{(\widetilde{\varphi}_a)}$$

  - Gains from trade: Gains for increase in variety + gains for improved productivity (models with variable mark-ups can display pro-competitive effects).