Seminario Avanzado de Comercio

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Problem Set Solution

1. *Derive the equilibrium conditions ZCP and FE in the closed economy. Find the equilibrium cutoff* φ^* *below which firms choose not to produce and the equilibrium mass of firms* M*.*

The pdf of the Pareto is:

$$g(\varphi) = \frac{kb^k}{\varphi^{k+1}}$$

So the average $\widetilde{\varphi}$ is:

$$\widetilde{\varphi}(\varphi^*) = \left[\int_{\varphi^*}^{\infty} \varphi^{\sigma-1}\mu(\varphi)\, d\varphi\right]^{\frac{1}{\sigma-1}}$$

where

$$\mu(\varphi) = \frac{k\varphi^{*k}}{\varphi^{k+1}}$$

$$\begin{aligned}
\widetilde{\varphi}(\varphi^*) &= \left[\int_{\varphi^*}^{\infty} \varphi^{\sigma-1}\frac{k\varphi^{*k}}{\varphi^{k+1}} d\varphi\right]^{\frac{1}{\sigma-1}} = \\
&= \left[k\varphi^{*k}\int_{\varphi^*}^{\infty} \frac{k+1-\sigma}{\varphi^{*(k+1-\sigma)}(k+1-\sigma)} \frac{\varphi^{*k+1-\sigma}}{\varphi^{k+1-\sigma+1}} d\varphi\right]^{\frac{1}{\sigma-1}} = \\
&= \left[\frac{k\varphi^{*k}}{\varphi^{*(k+1-\sigma)}(k+1-\sigma)}\int_{\varphi^*}^{\infty} (k+1-\sigma)\frac{\varphi^{*k+1-\sigma}}{\varphi^{k+1-\sigma+1}} d\varphi\right]^{\frac{1}{\sigma-1}} = \\
&= \left[\frac{k}{\varphi^{*(1-\sigma)}(k+1-\sigma)}\right]^{\frac{1}{\sigma-1}} = \varphi^*\left(\frac{k}{k+1-\sigma}\right)^{\frac{1}{\sigma-1}}
\end{aligned}$$

The ZPC condition is:

$$\begin{aligned}
\overline{\pi} &= f\left[\left(\frac{\widetilde{\varphi}}{\varphi^*}\right)^{\sigma-1} - 1\right] \\
\overline{\pi} &= f\left(\frac{k}{k+1-\sigma} - 1\right) \\
\overline{\pi} &= f\left(\frac{\sigma-1}{k+1-\sigma}\right)
\end{aligned}$$

The FE condition is:

$$\overline{\pi} = \delta f_e \left(\frac{\varphi^*}{b}\right)^k$$

To find φ^* we equate the two conditions:

$$\begin{aligned}\delta f_e \left(\frac{\varphi^*}{b}\right)^k &= f\left(\frac{\sigma-1}{k+1-\sigma}\right)\\ \varphi^* &= b\left[\frac{f}{\delta f_e}\left(\frac{\sigma-1}{k+1-\sigma}\right)\right]^{\frac{1}{k}}\end{aligned}$$

Thus average productivity $\widetilde{\varphi}\left(\varphi^*\right)$ is:

$$\widetilde{\varphi} = b\left[\frac{f}{\delta f_e}\left(\frac{\sigma-1}{k+1-\sigma}\right)\right]^{\frac{1}{k}}\left(\frac{k}{k+1-\sigma}\right)^{\frac{1}{\sigma-1}}$$

The mass of firms is given by:

$$\begin{aligned}M &= \frac{L}{\sigma\left(\overline{\pi}+f\right)}\\ &= \frac{L}{\sigma\left(f\left(\frac{\sigma-1}{k+1-\sigma}\right)+f\right)}\\ &= \frac{L\left(k+1-\sigma\right)}{\sigma f k}\end{aligned}$$

2. *Derive the equilibrium conditions ZCP and FE in an open economy with variable cost τ and fixed cost of exporting f_x per period (as we discussed in class it does not matter whether you assume a sunk cost f_{ex} or a per period f_x). Find the equilibrium cutoffs for domestic production and for exporting, φ^* and φ_x^*, and the mass of firms M. Compare the price index under autarky and trade. Can you conclude that welfare is always higher under trade than under autarky?*

Average profit is:

$$\overline{\pi} = \pi_d\left(\widetilde{\varphi}\right) + \frac{1-G\left(\varphi_x^*\right)}{1-G\left(\varphi^*\right)}\pi_x\left(\widetilde{\varphi}_x\right)$$

$$\overline{\pi} = \pi_d\left(\widetilde{\varphi}\right) + \left(\frac{\varphi^*}{\varphi_x^*}\right)^k \pi_x\left(\widetilde{\varphi}_x\right)$$

$$\begin{aligned}\overline{\pi} &= f\left[\left(\frac{\widetilde{\varphi}}{\varphi^*}\right)^{\sigma-1}-1\right] + \left(\frac{\varphi^*}{\varphi_x^*}\right)^k f_x\left[\left(\frac{\widetilde{\varphi}_x}{\varphi_x^*}\right)^{\sigma-1}-1\right]\\ \overline{\pi} &= f\left(\frac{\sigma-1}{k+1-\sigma}\right) + \left(\frac{\varphi^*}{\varphi_x^*}\right)^k f_x\left(\frac{\sigma-1}{k+1-\sigma}\right)\end{aligned}$$

where

$$\begin{aligned}\varphi_x^* &= \varphi^* \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \\ \frac{\varphi^*}{\varphi_x^*} &= \frac{1}{\tau}\left(\frac{f}{f_x}\right)^{\frac{1}{\sigma-1}}\end{aligned}$$

replace back and get

$$\begin{aligned}\overline{\pi} &= \left(\frac{\sigma-1}{k+1-\sigma}\right)\left[f + \left(\frac{\varphi^*}{\varphi_x^*}\right)^k f_x\right] \\ \overline{\pi} &= \left(\frac{\sigma-1}{k+1-\sigma}\right)\left[f + \frac{f_x}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k}{\sigma-1}}\right]\end{aligned}$$

The free entry condition is unaffected.

$$\overline{\pi} = \delta f_e \left(\frac{\varphi^*}{b}\right)^k$$

Equating the two conditions:

$$\begin{aligned}\delta f_e \left(\frac{\varphi^*}{b}\right)^k &= \left(\frac{\sigma-1}{k+1-\sigma}\right)\left[f + \frac{f_x}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k}{\sigma-1}}\right] \\ \varphi^* &= b\left[\frac{1}{\delta f_e}\left(\frac{\sigma-1}{k+1-\sigma}\right)\right]^{\frac{1}{k}}\left[f + \frac{f_x}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k}{\sigma-1}}\right]^{\frac{1}{k}} \\ \varphi^* &= b\left[\frac{f}{\delta f_e}\left(\frac{\sigma-1}{k+1-\sigma}\right)\right]^{\frac{1}{k}}\left[1 + \frac{1}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]^{\frac{1}{k}} \qquad (1)\end{aligned}$$

Since the average profit is higher in the open economy, the cutoff for domestic entry is higher. The mass of operating firms:

$$\begin{aligned}M &= \frac{L}{\sigma\left(\overline{\pi} + f + p_x f_x\right)} \\ &= \frac{L}{\sigma\left(\left(\frac{\sigma-1}{k+1-\sigma}\right)\left[f + \frac{f_x}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k}{\sigma-1}}\right] + f + \frac{1}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k}{\sigma-1}} f_x\right)} \\ &= \frac{(k+1-\sigma)L}{\sigma k\left[f + \frac{f_x}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k}{\sigma-1}}\right]} \\ &= \frac{(k+1-\sigma)L}{\sigma k f\left[1 + \frac{1}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]}\end{aligned}$$

The mass of varieties available to consumers:

$$\begin{aligned} M_t &= (1+p_x)\, M \\ &= \left[1+\frac{1}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k}{\sigma-1}}\right] \frac{(k+1-\sigma)L}{\sigma k f\left[1+\frac{1}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]} \end{aligned}$$

Now let's evaluate welfare in autarky and the open economy. Denote $\rho = \frac{\sigma-1}{\sigma}$. The price index depends on the mass of firms and the average productivity, so that real wage is equal to:

$$W = 1/P = M^{\frac{1}{\sigma-1}} \rho \widetilde{\varphi}$$

Real wage in autarky is (assume L=1):

$$\begin{aligned} W_a &= \left[\frac{(k+1-\sigma)}{\sigma f k}\right]^{\frac{1}{\sigma-1}} \rho b \left[\frac{f}{\delta f_e}\left(\frac{\sigma-1}{k+1-\sigma}\right)\right]^{\frac{1}{k}} \left(\frac{k}{k+1-\sigma}\right)^{\frac{1}{\sigma-1}} = \\ &= \left(\frac{1}{\sigma f_e}\right)^{\frac{1}{\sigma-1}} \rho b \left[\frac{1}{\delta}\left(\frac{\sigma-1}{k+1-\sigma}\right)\right]^{\frac{1}{k}} \end{aligned}$$

which corresponds to the expression for welfare on page 1721 of Melitz: $W_a = \rho\left(\frac{1}{\sigma f}\right)^{\frac{1}{\sigma-1}} \varphi_a^*$
In the open economy the real wage is equal to:

$$W = M_t^{\frac{1}{\sigma-1}} \rho \widetilde{\varphi}_t$$

average productivity is:

$$
\begin{aligned}
\widetilde{\varphi}_t &= \left[\frac{1}{M_t}\left(M\widetilde{\varphi}^{\sigma-1} + M_x\left(\frac{\widetilde{\varphi}_x}{\tau}\right)^{\sigma-1}\right)\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{M\left(1+p_x\right)}\left(M\widetilde{\varphi}^{\sigma-1} + p_x M\left(\frac{\widetilde{\varphi}_x}{\tau}\right)^{\sigma-1}\right)\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{1+p_x}\left(\widetilde{\varphi}^{\sigma-1} + p_x\left(\frac{\widetilde{\varphi}_x}{\tau}\right)^{\sigma-1}\right)\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{\left[1+\left(\frac{\varphi^*}{\varphi^*_x}\right)^k\right]}\left(\varphi^{*(\sigma-1)}\frac{k}{k+1-\sigma} + p_x\frac{\varphi_x^{*(\sigma-1)}}{\tau^{\sigma-1}}\left(\frac{k}{k+1-\sigma}\right)\right)\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{k}{(k+1-\sigma)\left[1+\left(\frac{\varphi^*}{\varphi^*_x}\right)^k\right]}\left(\varphi^{*(\sigma-1)} + \left(\frac{\varphi^*}{\varphi^*_x}\right)^k\frac{\varphi_x^{*(\sigma-1)}}{\tau^{\sigma-1}}\right)\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{k}{(k+1-\sigma)\left[1+\left(\frac{\varphi^*}{\varphi^*_x}\right)^k\right]}\left(\varphi^{*(\sigma-1)} + \varphi^{*k}\frac{\varphi^{*(\sigma-1-k)}\tau^{\sigma-1-k}(\frac{f_x}{f})^{\frac{\sigma-1-k}{\sigma-1}}}{\tau^{\sigma-1}}\right)\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{k}{(k+1-\sigma)\left[1+\left(\frac{\varphi^*}{\varphi^*_x}\right)^k\right]}\left(\varphi^{*(\sigma-1)} + \varphi^{*(\sigma-1)}\frac{1}{\tau^k}(\frac{f_x}{f})^{\frac{\sigma-1-k}{\sigma-1}}\right)\right]^{\frac{1}{\sigma-1}} \\
&= \left(\frac{k}{k+1-\sigma}\right)^{\frac{1}{\sigma-1}}\varphi^*\left[\frac{1+\frac{1}{\tau^k}(\frac{f}{f_x})^{\frac{k+1-\sigma}{\sigma-1}}}{1+\frac{1}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k}{\sigma-1}}}\right]^{\frac{1}{\sigma-1}}
\end{aligned}
$$

Putting together the two expressions we find real wages (again $L=1$):

$$
\begin{aligned}
W &= M_t^{\frac{1}{\sigma-1}}\rho\widetilde{\varphi}_t \\
&= \left[1+\frac{1}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k}{\sigma-1}}\right]^{\frac{1}{\sigma-1}}\left\{\frac{(k+1-\sigma)}{\sigma k f\left[1+\frac{1}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]}\right\}^{\frac{1}{\sigma-1}}\rho\left(\frac{k}{k+1-\sigma}\right)^{\frac{1}{\sigma-1}}\varphi^*\left[\frac{1+\frac{1}{\tau^k}(\frac{f}{f_x})^{\frac{k+1-\sigma}{\sigma-1}}}{1+\frac{1}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k}{\sigma-1}}}\right]^{\frac{1}{\sigma-1}} \\
&= \left(\frac{1}{\sigma f}\right)^{\frac{1}{\sigma-1}}\varphi^*\rho
\end{aligned}
$$

Gains from trade, $\frac{W_t}{W_a}$, depend only on the domestic cutoffs:

$$\frac{W}{W_a} = \frac{\left(\frac{1}{\sigma f}\right)^{\frac{1}{\sigma-1}} \varphi^* \rho}{\left(\frac{1}{\sigma f}\right)^{\frac{1}{\sigma-1}} \varphi_a^* \rho} = \frac{\varphi^*}{\varphi_a^*}$$

Since the domestic cutoff in the open economy is higher ($\varphi^* > \varphi_a^*$), real wages (and welfare) are higher ($W_t > W_a$).

3. *Starting from the open economy consider the effect of a reduction in τ on the two cutoffs φ^* and φ_x^*. How does the price index move with a decline in τ?*

From (1) the domestic cutoff is decreasing in τ. The exporting cutoff is:

$$\begin{aligned}
\varphi_x^* &= \varphi^* \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \\
&= b\left[\frac{f}{\delta f_e}\left(\frac{\sigma-1}{k+1-\sigma}\right)\right]^{\frac{1}{k}} \left[1 + \frac{1}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]^{\frac{1}{k}} \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \\
&= b\left[\frac{f}{\delta f_e}\left(\frac{\sigma-1}{k+1-\sigma}\right)\right]^{\frac{1}{k}} \left[\tau^k + \left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]^{\frac{1}{k}} \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}
\end{aligned}$$

so the exporting cutoff is increasing in τ. Since the price index (and real wages) depends only on the domestic cutoff, a decline in τ increases φ^* and real wages, and decreases φ_x^* and the price index.