Seminario Avanzado de Comercio

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Problem Set Solution

1. *Derive the equilibrium conditions ZCP and FE in the closed economy. Find the equilibrium cutoff* φ^* *below which firms choose not to produce and the equilibrium mass of firms* M.

The pdf of the Pareto is:

$$g\left(\varphi\right) = \frac{kb^k}{\varphi^{k+1}}$$

So the average $\widetilde{\varphi}$ is:

$$\widetilde{\varphi}\left(\varphi^*\right) = \left[\int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \mu\left(\varphi\right) d\varphi\right]^{\frac{1}{\sigma-1}}$$

where

$$\mu\left(\varphi\right) = \frac{k\varphi^{*k}}{\varphi^{k+1}}$$

$$\begin{aligned}
\widetilde{\varphi}\left(\varphi^*\right) &= \left[\int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{k\varphi^{*k}}{\varphi^{k+1}} d\varphi\right]^{\frac{1}{\sigma-1}} = \\
&= \left[k\varphi^{*k} \int_{\varphi^*}^{\infty} \frac{k+1-\sigma}{\varphi^{*(k+1-\sigma)}\left(k+1-\sigma\right)} \frac{\varphi^{*k+1-\sigma}}{\varphi^{k+1-\sigma+1}} d\varphi\right]^{\frac{1}{\sigma-1}} = \\
&= \left[\frac{k\varphi^{*k}}{\varphi^{*(k+1-\sigma)}\left(k+1-\sigma\right)} \int_{\varphi^*}^{\infty} \left(k+1-\sigma\right) \frac{\varphi^{*k+1-\sigma}}{\varphi^{k+1-\sigma+1}} d\varphi\right]^{\frac{1}{\sigma-1}} = \\
&= \left[\frac{k}{\varphi^{*(1-\sigma)}\left(k+1-\sigma\right)}\right]^{\frac{1}{\sigma-1}} = \varphi^* \left(\frac{k}{k+1-\sigma}\right)^{\frac{1}{\sigma-1}}
\end{aligned}$$

The ZPC condition is:

$$\begin{aligned}
\overline{\pi} &= f\left[\left(\frac{\widetilde{\varphi}}{\varphi^*}\right)^{\sigma-1} - 1\right] \\
\overline{\pi} &= f\left(\frac{k}{k+1-\sigma} - 1\right) \\
\overline{\pi} &= f\left(\frac{\sigma-1}{k+1-\sigma}\right)
\end{aligned}$$

The FE condition is:

$$\overline{\pi} = \delta f_e \left(\frac{\varphi^*}{b}\right)^k$$

To find φ^* we equate the two conditions:

$$\begin{aligned}
\delta f_e \left(\frac{\varphi^*}{b}\right)^k &= f\left(\frac{\sigma-1}{k+1-\sigma}\right) \\
\varphi^* &= b\left[\frac{f}{\delta f_e}\left(\frac{\sigma-1}{k+1-\sigma}\right)\right]^{\frac{1}{k}}
\end{aligned}$$

Thus average productivity $\widetilde{\varphi}(\varphi^*)$ is:

$$\widetilde{\varphi} = b\left[\frac{f}{\delta f_e}\left(\frac{\sigma-1}{k+1-\sigma}\right)\right]^{\frac{1}{k}}\left(\frac{k}{k+1-\sigma}\right)^{\frac{1}{\sigma-1}}$$

The mass of firms is given by:

$$\begin{aligned}
M &= \frac{L}{\sigma(\overline{\pi}+f)} \\
&= \frac{L}{\sigma\left(f\left(\frac{\sigma-1}{k+1-\sigma}\right)+f\right)} \\
&= \frac{L(k+1-\sigma)}{\sigma f k}
\end{aligned}$$

2. *Derive the equilibrium conditions ZCP and FE in an open economy with variable cost τ and fixed cost of exporting f_x per period (as we discussed in class it does not matter whether you assume a sunk cost f_{ex} or a per period f_x). Find the equilibrium cutoffs for domestic production and for exporting, φ^* and φ_x^*, and the mass of firms M. Compare the price index under autarky and trade. Can you conclude that welfare is always higher under trade than under autarky?*

Average profit is:

$$\overline{\pi} = \pi_d(\widetilde{\varphi}) + \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}\pi_x(\widetilde{\varphi}_x)$$

$$\overline{\pi} = \pi_d(\widetilde{\varphi}) + \left(\frac{\varphi^*}{\varphi_x^*}\right)^k \pi_x(\widetilde{\varphi}_x)$$

$$\begin{aligned}
\overline{\pi} &= f\left[\left(\frac{\widetilde{\varphi}}{\varphi^*}\right)^{\sigma-1}-1\right] + \left(\frac{\varphi^*}{\varphi_x^*}\right)^k f_x\left[\left(\frac{\widetilde{\varphi}_x}{\varphi_x^*}\right)^{\sigma-1}-1\right] \\
\overline{\pi} &= f\left(\frac{\sigma-1}{k+1-\sigma}\right) + \left(\frac{\varphi^*}{\varphi_x^*}\right)^k f_x\left(\frac{\sigma-1}{k+1-\sigma}\right)
\end{aligned}$$

where

$$\varphi_x^* = \varphi^* \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}$$
$$\frac{\varphi^*}{\varphi_x^*} = \frac{1}{\tau}\left(\frac{f}{f_x}\right)^{\frac{1}{\sigma-1}}$$

replace back and get

$$\overline{\pi} = \left(\frac{\sigma-1}{k+1-\sigma}\right)\left[f + \left(\frac{\varphi^*}{\varphi_x^*}\right)^k f_x\right]$$
$$\overline{\pi} = \left(\frac{\sigma-1}{k+1-\sigma}\right)\left[f + \frac{f_x}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k}{\sigma-1}}\right]$$

The free entry condition is unaffected.

$$\overline{\pi} = \delta f_e \left(\frac{\varphi^*}{b}\right)^k$$

Equating the two conditions:

$$\delta f_e \left(\frac{\varphi^*}{b}\right)^k = \left(\frac{\sigma-1}{k+1-\sigma}\right)\left[f + \frac{f_x}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k}{\sigma-1}}\right]$$
$$\varphi^* = b\left[\frac{1}{\delta f_e}\left(\frac{\sigma-1}{k+1-\sigma}\right)\right]^{\frac{1}{k}}\left[f + \frac{f_x}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k}{\sigma-1}}\right]^{\frac{1}{k}}$$
$$\varphi^* = b\left[\frac{f}{\delta f_e}\left(\frac{\sigma-1}{k+1-\sigma}\right)\right]^{\frac{1}{k}}\left[1 + \frac{1}{\tau^k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]^{\frac{1}{k}} \quad (1)$$

Since the average profit is higher in the open economy, the cutoff for domestic entry is higher.